The effect of income

on optimal two-part tariffs

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Abstract

Two-part tariffs are a common feature of utility pricing. In particular, telecommunications firms apply the two-part tariff to pricing telephony for which there is a telephone network access price and a telephone call price. In this paper the influence of income on an optimal two-part tariff is analysed for a range of market scenarios that includes maximising profit and maximising economic efficiency. The cost of the telephony service also consists two-parts: a constant marginal cost per subscriber and a constant marginal cost per call. The telephony service is offered to a population of individuals who have the same preferences for all goods and services but different incomes. The main finding is that if there is a positive income effect then the optimal two-part tariff call price is greater than the marginal cost of calling. Furthermore, the contribution to the firm as a result of the call price being greater than its marginal cost is transferred to the optimal access fee. This result holds for the range of market scenarios considered in this paper.

JEL Classification D11, D40

Introduction

The consumer outlay for many goods and services is determined by a two-part tariff. The two-part tariff typically consists of a fixed fee that allows consumption of the good or service and a price per unit of the good or service consumed. Frequently encountered examples of two-part tariffs include electricity supply, gas supply, telephony service, sport clubs, theme parks and internet services, just to name a few. These examples suggest that the two-part tariff is of practical significance and a common feature of pricing. This paper examines the optimal two-part tariffs for a telephone service. The derived results are expected to be of relevance for the other examples.

The number of factors that may determine whether an individual takes up a telephone service can potentially equal the number of individuals in a population. The need to isolate the most important factors dictate that preferences of all individuals in a population be treated as if they are the same. Income is not assumed to be the same across the population, thereby isolating it as the only factor that differentiates individuals' consumption choices. Other potential determinants that differentiate individual's consumption choice can be integrated into the framework developed in this paper, but this is left to a separate exercise.[1]

As well as explaining the effect of *income* on demand, the income effect offers a subtle explanation of the complementary demand between access and calling. For example, an increase (decrease) in the access price decreases (increases) the disposable income available for calling. Furthermore, an increase (decrease) in the call price decreases (increases) an individual's willingness to pay for access. The income effect is one factor that can influence the consumption outcome, due to the complementary relationship between access and calling. Not taking account of the income effect may lead to an implicit conclusion that there is no relationship, which results in the setting of economically inefficient tariffs.

Optimal two-part tariffs are determined for three market scenarios. These scenarios include the firm maximising its profit, a regulator maximising the sum of the consumer surplus and the firm's profit, and a regulator maximising the sum of the consumer surplus and the firm's profit subject to a constraint on the firm's profit. The purpose of the analysis is to understand how the firm and regulator, in their own interest, affect the optimal two-part tariff.

The paper has four parts. First, the income effect is defined and its treatment in the previous studies on two-part tariffs discussed. Second, a consumer choice model is developed for a telephone service. Third, market structures are proposed, based on the regulator's and the firm's objectives, and finally conclusions are drawn.

Income Effect

In this paper, individuals in a population are assumed to have the same preferences for goods and services but different incomes. Thus, income is isolated as a determinant of the individuals' consumption choice. As the name suggests, the income effect is the effect of a change in income on the demand for a good. The standard classification of the income effect, offered in economic texts such as Kreps (1990), describes the affected good as inferior, a necessity or a luxury. The demand for an inferior good decreases with an increase in income. The demand for a necessity either does not change or increases with demand by proportionally less than the necessity's expenditure share. The demand for a luxury increases with demand by proportionally more than the luxury's expenditure share. Two cases of the income effect of particular interest are a subset of the standard classifications. There is the case of a change in income not affecting demand, which is referred to as *no income effect*. Also, there is the case of an increase (decrease) in income resulting in a corresponding increase (decrease) in demand for the good, which is referred to as a *positive income effect*. The case of an inferior good is not of interest for this study.

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[1] An example of an additional determinant is the network externality. For a telephone network, a network externality is present if the existing subscribers benefit from an additional subscriber joining the telephone

The starting point in the literature for this paper is Coase's (1946) analysis of two-part tariffs. Coase describes a market in which the supply of a good is determined by a two-part cost. That is, there is a constant marginal cost per individual who consumes the good and there is a constant marginal cost per unit of good consumed. Coase argues that the optimal price structure for the good is a two-part tariff, with each price component set equal to the corresponding marginal cost. This paper supports Coase's conclusion, but under the restriction that there is no income effect. This restriction is relaxed and different market objectives are considered.

The theoretical treatment of the influence of the income effect on an optimal two-part tariff has been erratic. This seems peculiar given that the fixed component of the two-part tariff may directly affect an individual's expenditure on the usage of the particular service. It can be assumed, as do Ng and Wessier (1974), Goldman, Leland and Sibley (1984), Brown and Sibley (1986), that the preferences of consumers differ according to an index. They commence their analysis by assuming that there is no income effect. Then to consider the effect of the income effect on the optimal pricing problem, they include the income effect by substituting income for the index of preferences and derive an *indirect* utility function. However, the ad hoc inclusion of income can be interpreted as affecting *both* the individuals' preferences, described by the utility function, and the budget constraint. This ambiguity of the effect of income on the indirect utility function carries through to the derivation and interpretation of the results.

It is proposed here to start from first principles. It is assumed that all individuals in a population maintain the same preferences irrespective of income. Individual's preferences are not ordered according to some index. However, individuals' incomes are allowed to vary across the population. In this way the specific consequence of the income effect can be examined separately from consumers' individual behavioural idiosyncrasies. These idiosyncrasies could be included in the framework derived here as a separate exercise.

Casual observation suggests the demand for access to a telephone network is not independent of the usage associated with the access. It seems reasonable to assume individuals do not derive benefits from access to a telephone network in its own right, but they derive benefits from calling. This implies individuals do not purchase access if they do not make calls. Evans' (1996) observations of the market penetration of telephony among residential customers in New Zealand between 1987 and 1995 are consistent with this hypothesis. Evans notes, first, during this period there was a significant increase in the standard residential monthly rental for access, which was offset by a reduction in national and international toll-call prices. Evans proposes the increase in monthly line rental did not effect the penetration of telephony because it was more than offset by the reduction in toll-call prices. Secondly, during the early 1990s there was a

network.

reduction in government welfare payments to the unemployed, which it is proposed reduced the penetration of telephony among customers with a low income. These empirical observations are consistent with the hypothesis that the demand for telephony access is conditional on the demand for usage, and changes in income affects the demand for telephony.

The access decision implies that the marginal utility of income cannot be constant. A constant marginal utility of income implies the expenditure share between telephony and non-telephony goods is constant as income changes for a given set of prices. However, it is not possible to keep an individual's expenditure share constant if, with a change in income, there is a change from consuming only non-telephony goods to consuming telephony and non-telephony goods.

Oi (1971) considers a monopoly offering a service with an outlay determined by a two-part tariff. Oi's optimal usage-pricing proposition for a profit maximising monopoly is replicated here by imposing the restriction that there is no income effect. This analysis expands on Oi's study by considering a regulatory scenario and by specifically taking into account the income effect.

Ng and Weisser (1974) analyse a two-part tariff with licence fee and usage price. They are concerned with maximising consumer surplus subject to a constraint on the firm profits with the firm facing decreasing average costs. With this objective, Ng and Weisser propose, first, in the presence of a positive income effect the optimal usage price is greater than the usage marginal cost. Secondly, they propose if the firm's profit is constrained to be at least zero then the access fee will be greater than zero. This analysis agrees with Ng and Weisser's general propositions but not their rationale. Furthermore, Ng and Weisser do not consider the endogenous determination of the number of subscribers.

Schmalensee (1981) considers the optimal two-part tariff for a service with a constant marginal cost per subscriber, and in addition a constant marginal cost per unit usage. This is the same price and cost structure considered here. When there is no income effect, and given the objective of maximising consumer surplus subject to a profit constraint, Schamlensee shows the optimum usage prices equals the marginal cost of usage and the access fee equals the marginal cost of an additional subscriber. Schmalensee then includes the income effect in the analysis but restricts the marginal utility of income to a constant for all individuals who subscribe to the network. This restriction contradicts the proposition to analyse the access decision and the effect of the access fee on the subsequent usage decision. Schmalensee's restriction is relaxed in the following analysis.

Goldman, Leland and Sibley (1984) generalise the two-part tariff to consider non-uniform price schedules, where prices depend on the amount purchased, with no and a positive income effect. The objective of the analysis is to determine the optimal non-uniform price schedule that maximises consumer surplus subject to the profit constraint. Their main conclusion with respect to optimum price schedule, in the absence of an income effect, is that the increments of consumption should be priced according to the inverse Ramsey pricing rule. When the income effect is present the optimal pricing problem is equivalent to an optimal taxation that redistributes income. From the perspective of two-part tariff pricing Goldman et al's analysis is an extension on Ng and Weisser's analysis. As Ng and Weisser, Goldman et al do not consider the cost of providing service to an additional customer, which is considered here.

The span and depth of prior studies of two-part tariff theory reflects its wide practical significance. This study departs from the prior studies by explicitly considering the effect of the income effect on optimal two-part tariffs.

Consumer Behaviour

Simply being able to call someone on a telephone network is not sufficient to warrant subscribing to the network, especially if there is some cost or effort associated with subscription. The individual needs to actually expect to make calls, thus derive benefits, in order to justify subscription. Even if the individual subscribes to a network just for the option to make a call, there exists a certainty equivalent valuation for the option to make a call with a positive likelihood of making the call.[2] For example, an individual subscribes to a telephone network only to make emergency calls, say, in case of a fire. If the individual were certain that there would be no fire then they would not subscribe. Given a positive probability of this event, there exists a certainty equivalent valuation to make an emergency call and thus to subscribe to the network. This suggests the relatively self-evident proposition that network subscription is derived from network usage.

It is proposed that individuals derive utility from making telephone calls, x, and the consumption of non-telephony goods, z. It is assumed that all individuals have the same utility function:

$$U(z,x) \tag{1}$$

This utility function is twice continuously differentiable everywhere, is strictly increasing and is strictly quasi-concave.

It is assumed that:

[2] Refer to Kridel et.al. (1993) for a discussion on the option value of network access.

$$z > 0$$
$$x \geq 0$$

(2)

This assumption states it is necessary for an individual to consume some non-telephony goods, for example food, however the individual has the option to not make telephone calls.

If an individual's total income is y and they do not call, and assuming that a unit of the non-telephony good has a unit price, then an individual's utility is:

$$U(y,0)$$

(3)

It is assumed that the individual has other methods of communication than telephony. This is a necessary but not a sufficient condition for an individual to not subscribe to the telephone network. A result of the individual having the choice not to make telephone calls is their utility is not a homogeneous function of subscription to the telephone network and non-telephony goods. It follows that the marginal utility of income cannot be constant.

A necessary condition for an individual to choose to subscribe to the telephone network is:

$$U(y,0) \leq U(y-a, x)$$

(4)

where:

$$z = y - a$$

(5)

It is assumed that individuals are required to make consumption decisions based on an economic trade-off between calling and non-telephony goods. It follows that the variable a represents some value greater than or equal to zero, which the individual foregoes in order to subscribe to and call on the telephone network. Assuming some positive level of calling and income, the maximum value that a can take is the subscriber's willingness to pay for telephony. This is the maximum value of non-telephony goods that an individual is willing to forego in exchange for telephony service. The willingness to pay is represented by the variable w and is defined by:

$$U(y,0) \equiv U(y-w, x)$$

(6)

An individual will subscribe if the utility derived from the telephone calls and the non-telephony goods is greater than or equal to the utility derived from solely consuming non-telephony goods. The inclusion of the indifferent individuals, who will be referred to as marginal subscribers, in the group that subscribes is based on an assertion that these individuals will subscribe out of altruism for the infra-marginal subscribers. The marginal subscribers' altruism does not affect the non-subscribers as the marginal subscribers can continue to communicate with the non-subscribers but this communication is not by telephone. Only subscribers can communicate with each other by telephone.

A subscriber's outlay for telephony service is determined by a two-part tariff. There is a fixed monthly network access fee, f, and a price per call, p. Individuals are required to pay an access fee, f, before making telephone calls. An individual's total outlay for non-telephony goods plus telephony services is constrained by their income, such that:

$$y \geq z + p.x + f \tag{7}$$

A subscriber must satisfy both the budget constraint given by equation (7) and the constraint on the utility given by (4). Individuals who satisfy both these constraints, and therefore subscribe to the telephony service, will outlay no more than their willingness to pay for telephony, that is:

$$w \geq p.x + f \tag{8}$$

By definition, the consumer surplus is the value of the net benefit an individual derives from subscribing relative to not subscribing to the telephone network. It is the difference between an individual's willingness to pay for access given a positive level of calling and the outlay for telephony access and calling, that is:

$$cs \equiv w - p.x - f \tag{9}$$

Based on this definition and following on from equation (8) an individual will subscribe if:

$$cs \geq 0 \tag{10}$$

The relationship between income and consumer surplus is required for the subsequent analysis. An expenditure function, $e(p, U(y,0))$, is defined as a subscriber's minimum expenditure on calling plus non-telephony goods, which is required to achieve the utility equivalent to not subscribing, $U(y,0)$, given the call price p and the subscriber's income y. Under the assumptions regarding consumer preferences the value of $e(p, U(y,0))$ will always be less than or equal to y. The solution to this optimisation problem is:

$$e(p, U(y,0)) \equiv p \cdot x^* + z^* \tag{11}$$

Equating the consumption of non-telephony goods at the solution with it defined by a subscriber's willingness to pay – ie, equation (6) – then:

$$y - w = e(p, U(y,0)) - p \cdot x^* \tag{12}$$

Rearranging this term and substituting into equation (9) by eliminating the willingness to pay term gives the expression for the consumer surplus:

$$cs = y - e(p, U(y,0)) - f \tag{13}$$

This leads to a definition for the access fee. Assuming the marginal subscribers spend their total income y_i, the implied access fee for the marginal subscribers is determined by noting their consumer surplus equals zero and substituting this into equation (13) to derive:

$$f(p, y_i) = y_i - e(p, U(y_i, 0)) \tag{14}$$

The expression presents the access fee as a function of the marginal subscribers' income and the call price. An implicit boundary condition is that $f(p, y_i) \geq 0$ because it is not feasible for $e(p, U(y_i, 0))$ to be greater than y_i if $x \geq 0$ and $p > 0$.

The model specification has removed the possibility that the firm pays all individuals an incentive to subscribe with which they can spend on calling and non-telephony goods. The exclusion of a negative access fee is not a trivial assumption but reflects a fundamental aspect of access. A scenario can be envisaged that cannot be considered by the framework presented here, which is to allow the access fee to take any value less than zero – i.e., additional income – as well as greater than or equal to zero, for $x \geq 0$ and $p > 0$. It is obvious that if the access fee is negative all individuals will subscribe irrespective of making any calls, thus removing the need to consider access as a separate decision. This highlights the point that the economic problem with a negative access fee maybe fundamentally different than that for a non-negative access fee. In effect it goes beyond the efficiency of the telephony market *per se* to a wider question of solely the appropriate income distribution.

Having established a basic framework for analysing consumer behaviour, consideration turns to expanding this framework to include several market scenarios.

Market Scenarios

The optimum two-part tariff is determined for three market scenarios. The three scenarios are, first, a monopoly maximising profit, second, a regulator maximising the total market surplus defined by the sum of producer plus consumer surplus, and third, a variation on the second scenario that is a regulator maximising the total surplus but subject to a constraint on the firm's profits.

The scenarios can be characterised by the objective function:

$$\begin{aligned} W &= (CS + \Pi) \cdot b + (1 - b) \cdot \Pi \\ &= CS \cdot b + \Pi \end{aligned} \tag{15}$$

The parameter b is included to explicate the implications of scenarios for the optimal two-part tariff. If the scenario is to maximise the firm's profit then $b = 0$. If the scenario is to maximise a total surplus then $b = 1$. The objective function expresses the total market surplus, W, as the weighted sum of the aggregate consumer surplus function, CS, and the firm's profit function, Π.

The market aggregate consumer surplus for the telephone service is the sum of all subscribers' consumer surplus as defined by (13). The aggregate consumer surplus is:

$$CS = \int_{y_i}^{\infty} \left(y - e(p, U(y, 0)) - f(p, y_i)\right) \cdot m(y) dy \tag{16}$$

The term y_i is the income of the marginal individual and $m(y)$ is a uni-modal density function of income. The population size is assumed to be large enough to allow the function $m(y)$ to be treated as a continuous function that is differentiable between $0 < y < \infty$. A specific example of $m(y)$ is the log-normal distribution, but a specific functional form is not necessary to derive the propositions presented below. In fact, the uni-modal assumption is not necessary to derive the propositions. It is included to simplify the exposition by avoiding the necessity to consider multiple solutions. The definition of $m(y)$ is consistent with the assumption that income is the only characteristic that differentiates individuals in the population.

The profit function for the firm supplying the telephone service is:

$$\Pi = (p - c) \cdot X + \left(f(p, y_i) - g\right) \cdot M(y_i) \tag{17}$$

The price per minute of calling is p, the corresponding cost per minute of calling is c, the monthly rental price for customer access is the function $f(p, y_i)$ defined by equation (14), and the monthly cost for providing customer access is g.

The market total volume of telephone calls is:

$$X = \int_{y_i}^{\infty} x(p, f(p, y_i), y) \cdot m(y) dy$$

(18)

The function $x(p, f(p, y_i), y)$ is the demand for calling of an individual with income y if the call price is p and access fee is $f(p, y_i)$. The total number of telephone network subscribers is:

$$M(y_i) = \int_{y_i}^{\infty} m(y) dy$$

(19)

The first order conditions that maximise equation (15) with respect to the variables p and y_i, are:

$$\begin{aligned} W_p &= CS_p \cdot b + \left(X + (p-c) \cdot X_p + f_p(p, y_i) \cdot M(y_i)\right) = 0 \\ W_{yi} &= CS_{yi} \cdot b + \left((p-c) \cdot X_{yi} - (f-g) \cdot m(y_i) + f_{yi} \cdot M(y_i)\right) = 0 \end{aligned}$$

(20)

where[3]:

$$CS_p = h(p, U(y_i, 0)) \cdot M(y_i) - H$$

(21)

and[4],

---

3 $$\begin{aligned} CS_p &= \int_{y_i}^{\infty} \left(e_p(p, U(y,0)) - f_p(p, y_i)\right) \cdot m(y) dy \\ &= \int_{y_i}^{\infty} \left(-h(p, U(y,0)) + h(p, U(y_i, 0))\right) \cdot m(y) dy \\ &= h(p, U(y_i, 0)) \cdot M(y_i) - H \end{aligned}$$

4 $$\begin{aligned} CS_{yi} &= \int_{y_i}^{\infty} - f_{yi}(p, y_i) \cdot m(y) dy - \left(y_i - e(p, U(y_i, 0)) - f(p, y_i)\right) \cdot m(y_i) \\ &= - f_{yi}(p, y_i) \cdot M(y_i) \end{aligned}$$

$$CS_{y_i} = -f_{y_i}(p, y_i) \cdot M(y_i).$$

(22)

The function $h(p,U(y,0))$ is the compensated demand for calling if the individual indifferent between subscribing and not subscribing. The function $h(p,U(y,0))$ is determined by:

$$h(p,U(y,0)) = \frac{\partial e(p,U(y,0))}{\partial p}$$

(23)

The function H is the aggregate compensated demand for calling:

$$H = \int_{y_i}^{\infty} h(p,U(y,0)) \cdot m(y)dy$$

(24)

Substituting (22) and (21) into (20) and rearranging gives the mark-up for the call price and the access fee:

$$(p-c) = \frac{h(p,U(y_i,0)) \cdot M(y_i) \cdot (1-b) + H \cdot b - X}{X_p}$$

(25)

and:

$$(f(p,y_i) - g) = \frac{f_{y_i} \cdot M(y_i) \cdot (1-b) + (p-c) \cdot X_{y_i}}{m(y_i)}$$

(26)

The third market scenario is to maximise total surplus subject to a constraint on the firm's profits:

$$W = CS + \Pi$$

(27)

subject to,

$$p = \Pi$$

(28)

where the constraint, p , is prescribed by a regulator[5].

The Lagrangian function for the optimisation problem is:

$$L = CS + \Pi + l \cdot (\Pi - p) \tag{29}$$

The first order conditions optimising the Lagrangian function with respect to the variable p , y_i and the Lagrangian multiplier, l , are:

$$\begin{aligned} CS_{y_i} + (1 + l)\Pi_{y_i} &= 0 \\ CS_p + (1 + l)\Pi_p &= 0 \\ \Pi - p &= 0 \end{aligned} \tag{30}$$

The solution to (30) is equivalent to the solution to (20) if:

$$b = \frac{1}{1 + l} \tag{31}$$

Rearranging (30) in terms of the Lagrangian multiplier gives:

$$\begin{aligned} l &= -1 - \frac{CS_{y_i}}{\Pi_{y_i}} \\ l &= -1 - \frac{CS_p}{\Pi_p} \end{aligned} \tag{32}$$

The Lagrange multiplier can take positive values in a range from zero, which is equivalent to a non-binding constraint, through to infinity, which is equivalent to maximising profit without constraint. The value that the Lagrange multiplier takes depends on the value of p . As the regulator is assumed to be rational, p can take values between the profit resulting from maximising the total surplus and the profit if profit is maximised without constraint. The range of values that the Lagrange multiplier takes corresponds to the range for b as determined by (31). If $l = 0$ then $b = 1$, which is equivalent to maximising total surplus. If $l = \infty$ then $b = 0$, which is equivalent to maximising the firm's profit. If $0 < b < 1$ then the total surplus is maximised subject to a constraint on producer surplus. Furthermore, in this case b is interpreted

[5] Brown and Sibley (1986) present the same problem including an income effect. However, their solution implies that $H = X$, which contradicts their original assumption.

as a variable dependent on a constraint on profit by its relation to the Lagrange multiplier. The variable b is only interpreted as an independent variable if profit is not subject to a constraint, and then it can only take on values of 1 and 0 as defined.

Analysis of Results

The following propositions are derived from the model.

Proposition 1:

Under the assumptions made on preferences and the specific case of a positive income effect then:

$$c < p_{welfare} < p_{profit} \tag{33}$$

If there is no income effect, the corollary to this proposition is:

$$c = p'_{welfare} = p'_{profit} \tag{34}$$

The term p_{profit} is the optimal call price set by a monopoly maximising profit, and the term $p_{welfare}$ is the optimal call price set by a regulator maximising total surplus. The dash on the variable signifies there is no income effect.

Consider the profit maximising and total surplus maximising scenarios by respectively substituting $b = 0$ and $b = 1$ into equation (25) and rearranging the terms. The solution with respect to the call price for each scenario is:

$$p_{profit} = c + \frac{h(p, U(y_i, 0)) \cdot M(y_i) - X}{X_p} \tag{35}$$

$$p_{welfare} = c + \frac{H - X}{X_p} \tag{36}$$

If there is a positive income effect the terms to the right of the call cost in equations (35) and (36) meet the following conditions:[6]

[6] Refer to Appendix A for proof that $X_p < 0$.

$$
\begin{aligned}
h\left(p,U\left(y_i,0\right)\right).M\left(y_i\right) &< X \\
H &< X \\
h\left(p,U\left(y_i,0\right)\right).M\left(y_i\right) &< H \\
X_p &< 0
\end{aligned}
$$

(37)

These conditions imply the call price is greater than the call cost for both the profit maximising and total surplus maximising scenarios. This result can be seen by evaluating the sign of the term to the right of the call cost, c, in equations (35) and (36) and noting that the term is positive. Subtracting equations (36) from (35) gives:

$$
p_{profit} - p_{welfare} = \frac{h\left(p,U(y_i,0)\right)\cdot M\left(y_i\right) - H}{X_p}
$$

(38)

This indicates that $p_{welfare} < p_{profit}$ and that the difference between these prices is determined by the magnitude of the infra-marginal subscribers' income effect if there is a positive income effect.

The corollary to Proposition 1 is the case of no income effect. If there is no income effect the terms to the right of the call cost in equations (35) and (36) meet the following conditions:

$$
\begin{aligned}
h\left(p,U\left(y_i,0\right)\right).M\left(y_i\right) &= X \\
H &= X \\
h\left(p,U\left(y_i,0\right)\right).M\left(y_i\right) &= H \\
X_p &< 0
\end{aligned}
$$

(39)

Substituting these conditions into equations (35), (36) and (38), and evaluating the signs of the terms concludes that the optimal call price equals call cost for both profit and total surplus maximisation.

Comparing equation (36) with the Slutsky equation suggests an interesting interpretation of the ratio of the mark-up for calling over the call price, which is analogous to the Ramsey inversing pricing rule, that is:

$$
\frac{p_{welfare} - c}{p_{welfare}} = \frac{1}{h_H} - \frac{1}{h_X}
$$

(40)

The elasticity terms are defined by:

$$h_H = \frac{H}{p_{welfare}} \cdot \frac{\partial p_{welfare}}{\partial X}$$

(41)

$$h_H = \frac{X}{p_{welfare}} \cdot \frac{\partial p_{welfare}}{\partial X}$$

(42)

If there is a positive income effect, the main point to note is it is optimal to set the call price greater than cost for a wide range of cost and preference parameters. This condition holds for a general specification of preferences and income distribution. The magnitude of the margin between the call price and cost is determined by the magnitude of the income effect, as well as the regulator's and the firm's objectives.

The result for the total surplus maximising case is analytically equivalent to the result provided by Ng and Weisser (1974). Ng and Weisser proposed a case for the call price to be greater than the cost, but they did not explicitly relate this to the income effect as a possible cause. Furthermore, they did not carry the result through to determine the optimal access fee, which is considered next.

<u>Proposition 2:</u>

Under the assumptions made on preferences and the specific case of a positive income effect, then:

$$0 \le f_{welfare} < g$$

(43)

No general conclusion can be drawn regarding the magnitude of the access fee f_{profit} with respect to the access cost g .

If there is no income effect, the corollary to this proposition is:

$$f'_{welfare} = g$$

(44)

and ,

$$f'_{profit} > g$$

(45)

The terms f_{profit} and f'_{profit} are the optimal access fees for a monopoly maximising profit. The terms $f_{welfare}$ and $f'_{welfare}$ are the optimal access fees for a regulator maximising total surplus. The dash on the variable signifies that there is no income effect.

This proposition states that the total surplus maximising access fee is less than the access cost if there is an income effect. Under the total surplus maximising scenario the access fee is equal to cost if there is no income effect. It is uncertain whether the profit maximising access fee is greater than, less than, or equal to the access cost if there is a positive income effect. However, the profit maximising access fee is greater than the access cost if there is no income effect.

The total surplus maximising access fee is determined by substituting $b = 1$ into equation (26) and rearranging the terms to give:

$$f_{welfare} = g + \frac{(p_{welfare} - c) \cdot X_{y_i}}{m(y_i)} \tag{46}$$

As the access fee is a function of the call price, and the marginal subscribers' income, equations (36), (46) and (14) need to be solved simultaneously to determine the access fee, call price and the marginal subscribers' income. Furthermore, as implied by the definition of the access fee, equation (14), the sign of the access fee cannot be negative.

The sign of the terms to the right of the access cost g in equation (46) are considered in turn. Proposition 1 states that $(p_{welfare} - c) > 0$, if there is a positive income effect. In addition, it can be shown that $X_{y_i} < 0$ [7] holding p constant, because increasing the marginal income reduces the number of subscribers calling, and it also increases the access fee, which reduces the infra-marginal subscribers level of calling[8]. Evaluating the sign of the term to the right of the access cost concludes that the term is negative. It follows that the access fee is less than the access cost. Proposition 1 implies that the optimal access fee equals the access cost if there is no income effect under the welfare maximising scenario.

Comparing equations (36) with (46) suggests that there is a value transfer from calling to access, which keeps the access price below cost. The total value of the transfer is:

$$(p_{welfare} - c) \cdot \frac{X_{y_i}}{m(y_i)} \tag{47}$$

The value of the transfer is the product of the mark-up on calling with the sum of an individual marginal subscriber's level of calling plus the change in the infra-marginal subscribers' usage due to a change in the

[7] Refer to Appendices B and C for details.

access fee. The access fee reduction, due to the transfer, increases total calling by increasing the number of subscribers and the level of calling of the infra-marginal subscribers. The increase in calling resulting from a reduction in the access fee is offset by a reduction in calling due to the call price increase required to support the transfer. The value of the transfer is bounded by the access fee being non-negative, the particular market scenario and the population size.

The same value transfer from calling to access exists for the profit maximising scenario. Substituting $b = 0$ and rearranging equation (26) gives:

$$f_{profit} = g + \frac{f_{y_i} \cdot M(y_i) + (p_{profit} - c) \cdot X_{y_i}}{m(y_i)} \tag{48}$$

The transfer term, equation (47), is present in the above equation. The reduction in the access fee due to the transfer is offset by a marginal increase in the access fee revenue – ie, $f_{y_i} \cdot M(y_i)$ - which depends on the firm's ability to increase the access fee for the infra-marginal subscribers[9]. As a result it is unclear whether the access fee is greater than or less than the access cost. If there is no income effect the term $f_{y_i} \cdot M(y_i)$ is greater than zero and there is no transfer from calling., therefore the relationship described by equation (45) follows. Oi (1971) does not explicitly derive this result.

It is worth noting that the value transfer from the call price to the access fee indicates that the optimal two-part tariff takes on the role of an optimal redistributive taxation. The same transfer takes place for both the total surplus maximising and the profit maximising scenarios. Goldman, Leland and Sibley (1984) note a similar role for optimal non-linear tariffs if the income effect is positive.

Proposition 3:

> If a regulator maximises total surplus subject to a constraint on the firm's profits, as given by equations (27) and (28), then:
>
> $$c < p_{welfare} < p_{constrained} < p_{profit} \tag{49}$$
>
> and:
>
> $$0 \leq f_{welfare} < f_{constrained} < f_{profit}$$

---

[8] Refer to Appendix B for details.
[9] Refer to Appendix B for proof that $f_{y_i} > 0$ if the standard utility assumptions hold.

(50)

This proposition is conditional on the assumptions regarding consumer preferences, that the income effect is positive, and that the regulator selects a feasible profit constraint in the range:

$$p_{welfare} < p_{constrained} < p_{profit} \tag{51}$$

The optimal call price, $p_{constrained}$, and the optimal access fee, $f_{constrained}$, is the solution to maximising the total surplus subject to firm's profit constraint set equal to $p_{constrained}$. The profit constraint, $p_{constrained}$, is bounded within a range with a minimum value of $p_{welfare}$, which is the firm's profit if welfare is maximised without constraint, and a maximum value of p_{profit}, which is the firm's profit if it is maximised.

The Lagrange multiplier for the optimisation problem, equation (29), is:

$$l = \frac{p_{constrained} - p_{welfare}}{p_{profit} - p_{constrained}} \tag{52}$$

The Lagrange multiplier is bounded by the values that $p_{constrained}$ can take, therefore:

$$0 < l < \infty \tag{53}$$

Substituting this result into equation (31) determines the values that b can take, that is:

$$0 < b < 1 \tag{54}$$

Finally, evaluating equations (25) and (26) with respect to the implied dependency of b on $p_{constrained}$, and l, demonstrates propositions (49)and (50). Similarly, it can be inferred that Propositions 1 and 2 hold when maximising the total surplus subject to a profit constraint.

The focus of the analysis has been on determining the optimal two-part tariff if the income effect is positive or if there is no income effect. It is worth noting the implied market penetration of telephony for each case of income effect. Under the assumptions regarding consumer preferences and if there is no income effect then there cannot be both subscribers and non-subscribers simultaneously in the market. Either all individuals will subscribe or not subscribe, depending on the two-part tariff price points. This follows from the fact that if there is no income effect then income cannot serve as a factor that differentiates individuals' consumption choice, even though individuals do not have the same income. However, if the income effect is positive then both subscribers and non-subscribers can be simultaneously in the market.

The effect of a positive income effect on the number of subscribers can be illustrated by examining a pricing trajectory that moves away from a two-part tariff with prices equal to their respective marginal costs. The pricing trajectory can take the parametric form:

$$p = p^* \cdot t + c \cdot (1-t)$$
$$f = f^* \cdot t + g \cdot (1-t) \tag{55}$$

The parameter t is bounded such that $0 \le t \le 1$, p^* is the trajectory call price end point and f^* is the trajectory access fee end point. Substituting the parametric equation (55) in for the respective prices in the access fee equation (14) and differentiating the resulting implicit function with respect to t gives:

$$\frac{dy_i}{dt} = \frac{(p^* - c) \cdot h((p^* \cdot t + c \cdot (1-t)), U(y_i, 0)) + (f^* - g)}{f_{yi}} \tag{56}$$

This expression describes the marginal rate of change of the marginal subscribers' income with respect to a change in t along the linear two-part tariff trajectory from $p = c$ and $f = g$, to $p = p^*$ and $f = f^*$.

If $\frac{dy_i}{dt} > 0$ then the marginal subscribers' income is increasing, which implies a decrease in the number of subscribers. If $\frac{dy_i}{dt} < 0$ then the marginal subscribers' income is decreasing, which implies an increase in the number of subscribers. The expression states that the rate of change of a marginal subscriber's income equals the ratio of a marginal subscriber's contribution to the firm's profit over the marginal access fee with respect to the marginal subscriber. The expression implies that the number of subscribers will only increase above the case when prices equal costs if the marginal subscribers' contribution to the firm's profit is negative. This is because $f_{yi} > 0$[9].

Constraining firm's profit to zero implies that:

$$\frac{dy_i}{dt} = \frac{(p^* - c) \cdot \left(h((p^* \cdot t + c \cdot (1-t)), U(y_i, 0)) - \frac{X}{M} \right)}{f_{yi}} \tag{57}$$

The right hand term in brackets is the difference between a marginal subscriber's calls and the average calls for all subscribers. The sign of this term is negative because of the positive income effect. Equation (57) implies that if $p^* > c$ ($p^* < c$) then the market size will increase (decrease) while travelling the trajectory from c to p^*.

Considering the specific scenario where total surplus is maximised subject to the firm's profit being zero, it is shown Proposition 3 that then $p^* > c$. It follows then from equation (57) that the market size will increase along the trajectory described by equation (56). The market size will not continue to increase indefinitely as the call price is increased with the corresponding decrease in the access fee. There are three constraints on the market size: the first, trivial, constraint is that the market size cannot exceed the population size; the second constraint is the firm's objective to either maximise profit or restrict profit; and the third constraint is that the access fee cannot be negative. The constraint on the access fee implies that the decrease in the access fee, compensating for the increase in the call price, can only continue until the access fee is zero. The access fee will only equal zero for any further increase in the call price beyond this point. A further increase in the call price, with the access fee equal to zero, will reduce the consumer surplus and thus the market size. The reduction in consumer surplus is because there is no reduction in the access fee compensating the consumer surplus for the increase in the call price.

Conclusion

In this paper three propositions are presented regarding optimal two-part tariffs with respect to two specific types of income effect for a range of market scenarios. The types of income effect of interest are a *positive* income effect and *no* income effect on calling. It is assumed that consumer preferences follow a general specification on which restrictions are placed to examine the influence of the income effect on the optimal two-part tariff. The population of individuals, who can subscribe to the telephony service, have the same preferences but different incomes. It is assumed that income is distributed according to a uni-modal density function that is continuous and differentiable. Therefore, income is isolated and is the only determinant of consumer choice that is not the same among the individuals.

Consumers choose whether to access and if so how much to consume. A consequence of the access decision and access fee on an individual's preferences is that the marginal utility of income cannot be constant.

Three market scenarios are considered in this paper: these scenarios are a monopoly maximising profits, a regulator maximising the sum of the consumer surplus and the firm's profit, and a regulator maximising the sum of the consumer surplus and the firm's profit subject to a constraint on the firm's profit. The firm faces two costs to supply telephony, which are a cost per subscriber and a cost per call.

Having established the analytical framework, three propositions are presented. Considering the case of the positive income effect the propositions are as follows. First, the optimal call price is greater than the call cost for all scenarios. Second, there is a reduction in the access fee that is equivalent to the value of the mark-up on calling for all scenarios. Under the total surplus maximising scenario, a consequence of this transfer of value from calling to access is that the optimal access fee is below the access cost. Under the profit maximising scenario, the magnitude of the optimal access fee is indeterminate relative to the access cost. The value of this transfer depends on the extent of the income effect. Third, the optimal two-part tariff, under the scenario of maximising total surplus subject to any feasible constraint on the firm's profit, is bounded by the profit maximising optimal two-part tariff and the total surplus maximising two-part tariff.

The three corresponding propositions assuming that there is no income effect are as follows. First, the optimal call price is equal to the call cost for all scenarios. Second, there is no value transfer from calling to access because there is no income effect. It follows accordingly that under the total surplus maximising scenario that the optimal access fee is equal to the access cost. Under the profit maximising scenario the optimal access fee is greater than the access cost by an amount that captures the residual value of the consumer surplus. Third, under the scenario of maximising total surplus subject to any feasible constraint on the firm's profit, the feasible constraint on the firm's profit is at least zero. The constraint on profits acts to limit the amount of residual value of consumer surplus that is captured by the firm.

It is worth noting that if there is no income effect present and all individuals are the same except for income, then either all individuals will subscribe or they will not subscribe depending on the two-part tariff price points. If there is a positive income effect then it is feasible that not all individuals in a population will subscribe to the telephony service.

In summary, the presence, or not, of a positive income effect can significantly affect the form the optimal two-part tariff takes. Whether the income effect is positive or not is an empirical question. A set of hypotheses regarding the income effect, which can be tested against empirical data, can be derived from propositions and analytical framework presented in this paper. Any proposed set of hypotheses should encompass the relationship between income and expenditure on the service – telecommunications or electricity -, the relationship between income and the market penetration of telecommunications services, and the pricing of two-part tariffs relative to the cost of supply. Empirically testing these hypotheses will indicate the significance of the income effect and the consequential influence on the two-part tariff.

The theoretical model developed in the paper can expanded in two directions that are of practical relevance. The first direction is to include multiple two-part tariffs instead of the single tariff case considered in this paper. The second direction is to include the effect of a consumption externality: that is, subscribers benefit from the addition of new subscribers. It is envisaged that the model presented here can accommodate these developments.

Appendix A

Proposition A

$X_p < 0$ if the standard utility assumptions hold. This is unknown from the Slutsky equation and a positive income as the effect of the changing the call price on the access fee and consequently demand is unknown.

$$X = \int_{y_i}^{\infty} x(p, f(p, y_i), y) \cdot m(y) dy \tag{A. 1}$$

where;

$$X_p = \int_{y_i}^{\infty} \left(x_p \big|_{f,y} + x_f \big|_{p,y} \cdot f_p \right) \cdot m(y) dy \tag{A. 2}$$

and by the Slutsky Theorem;

$$x_p \big|_{f,y} = x_p \big|_u - x \cdot x_y \big|_{p,f} \tag{A. 3}$$

$$x_f \big|_{p,y} = -x_y \big|_{p,f} \tag{A. 4}$$

$$f_p = -e_p(p, U(y_i, 0)) = -h(p, y_i) \tag{A. 5}$$

Substituting into the bracketed term in the integral;

$$x_p \big|_u - x \cdot x_y \big|_{p,f} + x_y \big|_{p,f} \cdot h(p, y_i) \tag{A. 6}$$

or, by rearranging;

$$x_p \big|_u - x_y \big|_{p,f} \cdot (x - h(p, y_i)) \tag{A. 7}$$

This term is negative as:

$x_p \big|_u < 0$ due to the convexity assumption

$(x - h(p, y_i)) > 0$ if calling is normal

$x_y \big|_{p,f} > 0$ if there is a positive income effect

Note that it is not necessary for a positive income effect for this proposition to hold. If calling is inferior then $\left(x - h(p, y_i)\right) < 0$ and $x_y\big|_{p,f} < 0$, and the proposition holds.

Appendix B

The proposed that $f_{y_i} < 0$ if the standard assumptions regarding an individual's utility hold.

$$f\left(p, y_i\right) = y_i - e(p, U(y_i, 0)) \tag{B. 1}$$

Taking the partial derivative of $f\left(p, y_i\right)$ with respect to y_i:

$$f_{y_i}\left(p, y_i\right) = 1 - e_U(p, U(y_i, 0)) \cdot U_z(y_i, 0) \tag{B. 2}$$

It can be shown as a result of the convexity assumption that:

$$U_z(y_i, 0) < \frac{1}{e_U(p, U(y_i, 0))} \tag{B. 3}$$

for $p \leq p^*$, where:

$$y_i = e(p^*, U(y_i, 0)) \tag{B. 4}$$

Appendix C

The proposed that $X_{y_i} < 0$ if there is a positive income effect.

Starting with,

$$X = \int_{y_i}^{\infty} x(p, f(p, y_i), y) \cdot m(y) dy$$

(C. 1)

and taking the partial derivative with respect to the income of the marginal subscriber, then;

$$X_{y_i} = -x(p, f, y_i) \cdot m(y_i) + \int_{y_i}^{\infty} x_f \cdot f_{y_i} \cdot m(y) dy$$

(C. 2)

If a marginal individual subscribes then by definition;

$$x(p, f, y_i) > 0$$
$$m(y_i) > 0$$
$$M(y_i) > 0$$

(C. 3)

Furthermore, it can be shown that:

$$-x_f(p, f, y_i) = x_y(p, f, y)$$

(C. 4)

Taking this result and the assertion of a positive income effect implies that:

$$x_f(p, f, y_i) < 0$$

(C. 5)

Substituting the sign of these terms and noting that $f_{y_i} < 0$, as demonstrated in Appendix B, into the partial derivative of total calling with respect to the income of the marginal subscriber and evaluating demonstrates Proposition B2.

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